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Fluctuations of the quark densities in nuclei

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Abstract. We study the static scalar susceptibility of the nuclear medium, *i.e.*, the change of the quark condensate for a small modification of the quark mass. In the linear sigma model it is linked to the in-medium sigma propagator and its magnitude increases due to the mixing with the softer modes of the nucleon-hole excitations. We show that the pseudoscalar susceptibility, which is large in the vacuum, owing to the smallness of the pion mass, follows the density evolution of the quark condensate and thus decreases. At normal nuclear matter density the two susceptibilities become much closer, a partial chiral symmetry restoration effect as they become equal when the full restoration is achieved.

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1 Introduction

The problem of the quark condensate and chiral symmetry restoration in a dense medium has been extensively addressed. Little attention instead has been given to the question of the fluctuations and correlations of the quark scalar density. However for what concerns the chiral phase transition this quantity is as relevant as the evolution of the quark condensate. In the case of a second-order phase transition, large spontaneous fluctuations occur with an increase of the correlation length and of the relaxation time.

Nuclear matter at normal density is well below the critical density. Nevertheless chiral symmetry is appreciably restored. The order parameter decreases, as compared to its vacuum value, by $\simeq 35\%$. One may then wonder whether there exist also large spontaneous fluctuations of the quark scalar density. The present work addresses these questions. Our investigation is performed in an effective theory, the linear sigma model. The two fields introduced in this model to insure chiral symmetry, the pion and the sigma, have a great relevance in nuclear physics, in such a way that we can use, as a source of information, the experience acquired in this field. We investigate in particular the correlations of the quark scalar density, showing how their range increases with density. We also introduce the fluctuations of the pseudoscalar quark density. We investigate how it evolves with density and its relation to the order parameter.

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2 Scalar susceptibility

For an infinite system which possesses translational invariance the quark density correlator only depends on the relative space-time separation x. It can be defined as the retarded Green's function $\langle -i\Theta(x^0)\left[\overline{q}q(x),\overline{q}q(0)\right]\rangle$. Previous investigations [1,2] have addressed the question of the in-medium four-quark condensate, *i.e.*, the quantity $\langle\overline{q}q(x)\overline{q}q(x)\rangle$ which is a correlator taken at the same space-time point.

In the linear sigma model the symmetry-breaking piece of the Lagrangian is proportional to the sigma field:

$$\mathcal{L}_{\chi\mathrm{SB}} = c\,\sigma \tag{1}$$

with $c = f_\pi\, m_\pi^2$. This quantity plays the role of the symmetry-breaking Lagrangian of QCD:

$$\mathcal{L}_{\chi\mathrm{SB}}^{\mathrm{QCD}} = -2\, m_q\, \bar{q}\, q\,, \tag{2}$$

where $\bar{q}q = (\bar{u}u + \bar{d}d)/2$, $m_q = (m_u + m_d)/2$ and we have neglected isospin violation. Making use of the Gell-mann-Oakes-Renner relation, we obtain the following correspondence between the QCD and effective theory correlators:

$$\frac{\langle\bar{q}q(x)\,\bar{q}q(0)\rangle}{\langle\bar{q}q\rangle_{\mathrm{vac}}^2} = \frac{\langle\sigma(x)\,\sigma(0)\rangle}{f_\pi^2}\,, \tag{3}$$

where $\langle qq\rangle_{\mathrm{vac}}$ is the vacuum value of the condensate. The fluctuations of the quark density are thus carried by the sigma field, the chiral partner of the pion. The in-medium propagation of the sigma in the energy domain near the two-pion threshold, has been the object of several investigations (see, *e.g.*, [3–5]). Here we will focus on aspects which have been ignored, namely the low-energy region, below the particle-hole excitation energies.

The increase in the range of the correlator is reflected in the increase of the static susceptibility which can become divergent at the critical density in case of a second-order phase transition, a consequence of the appearance of a soft scalar mode [6,7]. In QCD the conjugate variables are the quark scalar density, which is the order parameter, and the quark mass, which is the exciting field, analogous to the magnetic field. As an amusing illustration of this analogy, in the Nambu-Jona-Lasinio (NJL) model, the constituent quark mass differs from the current one by the effect of the interaction with the condensate:

$$M_q = m_q \, - \, 2\, G_1 \, \langle \bar{q} q \rangle \, . \tag{4}$$

This relation presents an analogy with that between the magnetic field inside the ferromagnet and the applied one in the Weiss theory of magnetism:

$$\mathbf{H} = \mathbf{H}_0 \, + \, \lambda \, \mathbf{M} \, . \tag{5}$$

These two quantities differ by the existence of an internal field, $\lambda\,\mathbf{M}$, proportional to the magnetization $\mathbf{M}$. In NJL, the action of the quark condensate on the quark mass thus plays the role of the internal field of the Weiss theory.

The scalar susceptibility of QCD per unit volume represents the modification of the order parameter, which is the quark condensate, to a small perturbation of the quark mass, the parameter responsible for the explicit symmetry breaking:

$$\chi_{\mathrm{S}} = \frac{\partial \langle \bar{q} q \rangle}{\partial m_q} = 2 \int \mathrm{d}t' \, \mathrm{d}\mathbf{r}' \, G_{\mathrm{R}}(\mathbf{r} = 0, t = 0 \, , \, \mathbf{r}', t') \, , \tag{6}$$

where G_{R} is the retarded quark scalar correlator:

$$G_{\mathrm{R}}(\mathbf{r}, t, \mathbf{r}', t') = \Theta(t - t') \, \langle -i \big[\bar{q} q(\mathbf{r}, t) \, , \, \bar{q} q(\mathbf{r}', t') \big] \rangle \, . \tag{7}$$

The susceptibility represents space and time integrated correlators. In the effective theory, the quark density fluctuations are carried by the sigma field and the corresponding scalar static susceptibility is given by

$$\begin{aligned} \chi_{\mathrm{S}} &= \, 2 \, \frac{\langle \bar{q} q \rangle_{\mathrm{vac}}^2}{f_\pi^2} \int_0^\infty \mathrm{d}\omega \left(\frac{2}{\pi \omega} \right) \mathrm{Im} D_{\mathrm{SS}}(\mathbf{q} = 0, \omega) \\ &= \, 2 \, \frac{\langle \bar{q} q \rangle_{\mathrm{vac}}^2}{f_\pi^2} \, \mathrm{Re} D_{\mathrm{SS}}(\mathbf{q} = 0, \omega = 0) \, , \end{aligned} \tag{8}$$

where $D_{\mathrm{SS}}(\mathbf{q}, \omega)$ is the Fourier transform of the scalar correlator:

$$\begin{aligned} D_{\mathrm{SS}}(\mathbf{q}, \omega) = & \int \mathrm{d}t \, \mathrm{d}\mathbf{r} \, e^{i \omega t} \, e^{-i \mathbf{q} \cdot \mathbf{r}} \\ & \times \left\langle -i \, T \Big(\sigma(\mathbf{r}, t) - \langle \sigma \rangle \, , \, \sigma(0) - \langle \sigma \rangle \Big) \right\rangle \, . \end{aligned} \tag{9}$$

We have replaced the retarded Green's function by the time-ordered one which is identical for positive frequencies. One can also define a momentum-dependent susceptibility according to

$$\chi_{\mathrm{S}}(\mathbf{q}) = 2 \, \frac{\langle \bar{q} q \rangle_{\mathrm{vac}}^2}{f_\pi^2} \, \mathrm{Re} D_{\mathrm{SS}}(\mathbf{q}, \omega = 0) \, . \tag{10}$$

The range of the fluctuations of the quark density is thus given by the one of the sigma field. Notice that in the phase of broken symmetry, which is the case at ordinary densities, these fluctuations correspond to longitudinal ones, *i.e.*, along the direction of the spontaneous ordering which is that of the scalar field. In a very simple picture where the sharp sigma mass is reduced in the nuclear medium, *i.e.* m_σ replaced by some dropped value, m_σ^*, as has been suggested by Hatsuda *et al.* [4], the static correlator is $\exp(-m_\sigma^* r)/r$. Hence, as m_σ^* goes to zero at full chiral symmetry restoration, the fluctuations acquire an infinite range as for fluids near the critical temperature. In the work of Hatsuda *et al.* the sigma mass modification arises from the tadpole term. Here, we include the coupling of the sigma to the nucleon-hole excitations, which modifies the scalar field propagator as follows:

$$D_{\mathrm{SS}} = D_{\mathrm{S}}^0 \, (1 \, + \, D_{\mathrm{S}}^0 \, g_{\mathrm{S}}^2 \Pi_{\mathrm{SS}}) \, . \tag{11}$$

Here $D_{\mathrm{S}}^0 = 1/(q^2 - m_\sigma^2)$ is the bare sigma propagator, Π_{SS} is the full scalar NN^{-1} polarization propagator, and g_{S} is the σNN coupling constant. This last quantity is a function of the momentum transfer but it may also be density dependent, as proposed in the quark-meson coupling model (QMC) [8]. We will come back to this question. In the low-density limit where the non-relativistic approximation applies, the polarization propagator Π_{SS} reduces to the free Fermi gas expression:

$$\begin{aligned} \mathrm{Re} \Pi^0(\mathbf{q}, \omega = 0) = & - \frac{M_N \, k_{\mathrm{F}}}{\pi^2} \\ & \times \left[1 + \frac{1}{t} \left(1 - \frac{t^2}{4} \right) \ln \left(\frac{1 + t/2}{1 - t/2} \right) \right] \, . \end{aligned} \tag{12}$$

At low momentum, it is a slowly varying function of $t = |\mathbf{q}|/k_{\mathrm{F}}$:

$$\mathrm{Re} \Pi^0(\mathbf{q}, \omega = 0) \simeq \, - \frac{2 \, M_N \, k_{\mathrm{F}}}{\pi^2} \left(1 - \frac{t^2}{12} \right) \, . \tag{13}$$

This expression, taken at $t = 0$, introduced in the relation (11), provides the low-density expression for the unit volume scalar susceptibility of the infinite nuclear medium:

$$\begin{aligned} \chi_{\mathrm{S}} = 2 \, \frac{\langle \bar{q} q \rangle_{\mathrm{vac}}^2}{f_\pi^2} \, \mathrm{Re} D_{\mathrm{SS}}(\mathbf{q} = \mathbf{0}, \omega = 0) \simeq & \\ \chi_{\mathrm{S,vac}} \left(1 \, + \, \frac{2 \, g_{\mathrm{S}}^2 \, M_N \, k_{\mathrm{F}}}{\pi^2 \, m_\sigma^2} \right) \, . & \end{aligned} \tag{14}$$

Notice that, in our definition, the scalar susceptibility is, as the quark condensate, negative, *i.e.*, it increases the magnitude of the condensate. An in-medium increase of its absolute value thus opposes the restoration effect. For the actual evaluation of the RPA correction in the lowest order, we take the parameters from Guichon *et al.* [9]: $g_{\rm S}^2/m_\sigma^2 = 0.71\,{\rm GeV}^{-2}$. This estimate leads to a large modification of the sigma propagator: at normal nuclear matter density ρ_0, $D_{\rm S} \sim 12\,D_{\rm S}^0$. Thus we have reasons to believe in a sizeable in-medium increase of the scalar susceptibility. This reflects the increase of the range of the scalar quark density correlator: the quark density fluctuations, transmitted by the sigma-meson, are relayed by the nucleons, thus increasing their range. Notice that this effect can be interpreted as arising from an in-medium decrease of the sigma mass. This mass is a screening one and not the energy of the pole of the sigma propagator at zero momentum. We point out that the mixing with the NN^{-1} excitations does not affect the scalar spectral function at high energy, beyond the two-pion threshold, since $\Pi_{\rm SS}$ vanishes in this energy range, which is well above the nuclear excitation energies.

Now, for a more quantitative evaluation, we have to know how much the full $\Pi_{\rm SS}$ deviates from the free one: Π_0, which is a pure nuclear physics problem, on which experimental information can be obtained. Especially at normal nuclear matter density the full polarization propagator $\Pi_{\rm SS}$, at zero momentum, is linked to the incompressibility factor K of nuclear matter, the magnitude of which is known, even if the exact value is still under investigation:

$$\Pi_{\rm SS}(\mathbf{q}=0,\omega=0,\rho_0) \;=-\,\frac{9\,\rho_0}{K}\,. \tag{15}$$

With the currently suggested value, $K = 230$ MeV, which is practically the free Fermi gas one: $K = 3k_{\rm F}^2/M_N$, the quantity $\Pi_{\rm SS}(\mathbf{q}=0,\omega=0)$ also has the free Fermi gas value given in eq. (13). This supports the previous first-order estimate, which led to a large increase of the scalar susceptibility.

However, to be more precise, we have to discuss also in more details the value of the σNN coupling constant entering the renormalization factor in eq. (11). We mentioned that this quantity may depend on the density. This is the case in the quark-meson coupling model (QMC) [8]. This model preserves chiral invariance and its main consequence can be incorporated in the linear sigma model. It has been introduced to account for the nuclear saturation, which is not accounted for in the original sigma model [10]. In view of its relevance in our problem, we enter in some details of this model. It is an extension of the quantum hadrodynamics model [11,12], but formulated directly at the quark level. In QMC the scalar meson and vector mesons couple directly to the quarks inside the nucleons, described by a bag model. The crucial ingredient of the model is that the nucleon has an internal structure which adjusts to the presence of the scalar field. Indeed, under the influence of this attractive field the quark mass is lowered according to: $m_q^* = m_q - g_\sigma^q\langle\sigma\rangle$, where g_σ^q is the sigma quark coupling constant. Accordingly, the valence quark scalar number, which depends on the quark mass, also decreases, the quarks becoming more relativistic. This effect is directly related to the QCD scalar susceptibility of the bag, $\chi_{\rm S}^{\rm bag}$. Introducing the scalar charge, $Q_{\rm S}$, defined as the valence quark scalar number:

$$\begin{aligned} Q_{\rm S}(\langle\sigma\rangle) &= \int {\rm d}\mathbf{r}\,\left(\langle\bar{q}q(\mathbf{r})\rangle \,-\, \langle\bar{q}q\rangle_{\rm vac}\right)\\ &= Q_{\rm S}(m_q) \,+\, \chi_{\rm S}^{\rm bag}\,(m_q^* \,-\, m_q)\\ &= Q_{\rm S}(\langle\sigma\rangle=0) \,-\, g_\sigma^q\,\chi_{\rm S}^{\rm bag}\,\langle\sigma\rangle\,. \end{aligned} \tag{16}$$

Now, the scalar charge acting as the source of the scalar field, a decrease of the scalar charge amounts to a lowering of the σNN coupling constant, when the mean scalar field, *i.e.*, the density, increases:

$$g_{\rm S}(\rho) = g_\sigma^q\,Q_{\rm S}(\langle\sigma\rangle)\,. \tag{17}$$

In QMC this mechanism is responsible for the saturation of nuclear matter. Alternately the effect can be viewed as the creation of an induced scalar charge, $Q_{\rm S}^{\rm ind}$, due to the presence of the scalar field and proportional, for small intensities, to the field, with

$$Q_{\rm S}^{\rm ind} = -\,g_\sigma^q\,\chi_{\rm S}^{\rm bag}\,\langle\sigma\rangle, \qquad \text{per nucleon.} \tag{18}$$

In the nuclear medium, the presence of the induced charges modifies the propagation equation of the scalar field. Adding the intrinsic and the induced sources, we have in the static situation the following result in a uniform medium:

$$\langle\sigma\rangle = \frac{g_{\rm S}\,\rho_{\rm S}}{m_\sigma^2 \,+\, (g_\sigma^q)^2\,\chi_{\rm S}^{\rm bag}\,\rho_{\rm S}}\,. \tag{19}$$

Here $\rho_{\rm S}$ is the nucleon scalar density and $g_{\rm S}$ is the value of the coupling constant for a vanishing scalar field. The introduction of $\chi_{\rm S}^{\rm bag}$, which is positive amounts to an increase of the sigma mass:

$$m_\sigma^{*2}(\rho) = m_\sigma^2 \,+\, (g_\sigma^q)^2\,\chi_{\rm S}^{\rm bag}\,\rho_{\rm S}\,, \tag{20}$$

which counteracts the decrease due to the mixing with NN^{-1} states. Interpreted as an optical potential for the propagation of the sigma, related to the σN amplitude, this internal structure effect represents a non-Born σN amplitude. What we had discussed previously was the influence of the Born amplitude (no excitation of the nucleon took place) taken alone on the sigma propagation. We have now to combine the two influences, of the non-Born and Born, with the result:

$$D_{\rm S} \,=\tilde{D}_{\rm S}^0\,\left(1\,+\,g_{\rm S}^2\,\tilde{D}_{\rm S}^0\,\Pi_{\rm SS}\right) \tag{21}$$

with $\tilde{D}_{\rm S}^0 = -(m_\sigma^{*2}(\rho))^{-1}$, at zero four-momentum. Comparing with the vacuum value $D_{\rm S}^0$, we have at normal nuclear matter density

$$\begin{aligned} D_{\rm S} = \frac{m_\sigma^2}{m_\sigma^{*2}}\,D_{\rm S}^0\,\left(1\,+\,\frac{g_{\rm S}^2\,\rho_0}{m_\sigma^{*2}}\,\frac{9}{K}\right) &=\\ \frac{m_\sigma^2}{m_\sigma^{*2}}\,D_{\rm S}^0\,\left(1\,+\,g_{\rm S}\,\langle\sigma(\rho_0)\rangle\,\frac{9}{K}\right)&\,, \end{aligned} \tag{22}$$

where $\langle\sigma(\rho)\rangle$ is the average sigma field in the medium. For a bag radius of 0.8 fm Guichon *et al.* [9] give $g_{\rm S}\langle\sigma(\rho_0)\rangle \simeq$ 200 MeV. The corresponding increase of the square sigma mass is 18%. For consistency we use also the value of the model for the incompressibility: $K = 280$ MeV (also compatible with the experimental allowed range). With these values, the enhancement factor of the scalar susceptibility turns out to be $\chi_{\rm S}(\rho_0)/\chi_{\rm S,vac} = 6.2$, still a large medium effect. This increase reflects that of the range of the quark scalar density fluctuations which, transmitted by the sigma field, are relayed by the nucleons.

3 Pseudoscalar susceptibility

This factor applies to the "parallel" susceptibility, along the order parameter. In the broken phase, there exists a transverse one, along the "perpendicular" direction. For QCD this is the pseudoscalar susceptibility, linked to the fluctuations of the pseudoscalar quark density. We define it in such a way that it coincides with the scalar susceptibility in the restored phase:

$$\chi_{\rm PS} = 2\int {\rm d}t'\,{\rm d}\mathbf{r}'\,\Theta(t-t') \times \left\langle -i\left[\bar q\, i\gamma_5\,\frac{\tau_\alpha}{2}\,q(0)\,,\,\bar q\, i\gamma_5\,\frac{\tau_\alpha}{2}\,q(\mathbf{r}'\ t')\right]\right\rangle . \quad (23)$$

This pseudoscalar susceptibility is related to the correlator of the divergence of the axial current since

$$\partial^\mu \mathcal{A}^\alpha_\mu(x) = 2\,m_q\,\bar q\, i\gamma_5\,\frac{\tau_\alpha}{2}\,q(x)\,. \quad (24)$$

In representations where PCAC holds, which is the case in the linear sigma model or in specific representations of the non-linear one, the interpolating pion field is taken to be proportional to the divergence of the axial current according to

$$\partial^\mu \mathcal{A}^\alpha_\mu(x) = f_\pi\, m_\pi^2 \Phi^\alpha(x)\,. \quad (25)$$

The pseudoscalar susceptibility $\chi_{\rm PS}$ is then linked to the pion propagator, taken at zero momentum and energy:

$$\chi_{\rm PS} = \frac{f_\pi^2\, m_\pi^4}{2\,m_q^2}\int {\rm d}t'\,{\rm d}\mathbf{r}'\,\Theta(t-t')\langle -i\left[\Phi^\alpha(0)\,,\,\Phi^\alpha(\mathbf{r}'\ t')\right]\rangle = \frac{f_\pi^2\, m_\pi^4}{2\,m_q^2}{\rm Re}D_\pi(\mathbf{q}=0,\omega=0)\,. \quad (26)$$

Since the factor multiplying the pion propagator can be written as $2\,\langle\bar qq\rangle^2_{\rm vac}/f_\pi^2$, this equation is the analog of the one for the scalar susceptibility, with the pion replacing the sigma. Thus one reaches, in the linear sigma model, a symmetric situation where the two susceptibilities are governed by the propagators of the two chiral partners, σ and π. In the medium we denote $S(\mathbf{q}\,,\,\omega)$ (which implicitly depends on the density) the pion self- energy so that

$$D_\pi(\mathbf{q}=0,\omega) = \left[\omega^2 - m_\pi^2 - S(\mathbf{q}=0,\omega)\right]^{-1}. \quad (27)$$

The expression of the self-energy depends on the representation. The one that should enter here is the one which applies to the PCAC representation. Its expression which has been discussed by Delorme *et al.* [13] is not simple. Firstly because the π-N scattering amplitude itself has a complicated off-shell dependence, with the sign change between the Cheng-Dashen and the soft point. In addition, to second order in the nucleon density, it contains terms which are specific to this representation and are imposed by chiral symmetry. This complexity is however irrelevant for our purpose which is to establish a link with the condensate evolution. At zero four-momentum one has

$${\rm Re}D_\pi(\mathbf{q}=0,\omega=0) = -\frac{1}{m_\pi^2 + S(0,0)}\,. \quad (28)$$

On the other hand, the evolution with density of the condensate is governed by the nuclear sigma commutator: Σ_A/A per nucleon, according to the exact expression

$$\frac{\langle\bar qq(\rho)\rangle}{\langle\bar qq\rangle_{\rm vac}} = 1 - \frac{(\Sigma_A/A)\,\rho}{f_\pi^2\,m_\pi^2}\,. \quad (29)$$

The nuclear sigma commutator, which *a priori* depends on the density, is defined as the expectation value over the nuclear ground state of the commutator between the axial charge and its time derivative. From QCD this quantity is also the volume integral of the difference between the in-medium condensate and its vacuum value is

$$\Sigma_A = 2\,m_q\int {\rm d}\mathbf{r}\left(\langle\bar qq(\rho)\rangle - \langle\bar qq\rangle_{\rm vac}\right), \quad (30)$$

hence the relation (29). In the PCAC representation, the nuclear sigma commutator also represents the scattering amplitude for soft pions on the nuclear medium, $T(0,0)$ (per unit volume), with

$$\frac{(\Sigma_A/A)\,\rho}{f_\pi^2} = T(0,0)\,. \quad (31)$$

This last quantity is related to the pion self-energy, $S(q,\omega)$ through

$$T(0,0) = \frac{S(0,0)}{1 + S(0,0)/m_\pi^2}\,. \quad (32)$$

In this expression the denominator represents the coherent rescattering of the soft pion [14]. This distortion factor is needed in order to make the nuclear sigma commutator independent of the representation, as was discussed in ref. [13]. In fact, in the density dependence of the condensate, the distortion is cancelled by many-body terms of the self-energy but here we do not need the explicit form of the self-energy and it is necessary to include the distortion. It is now possible to establish a link between the pseudoscalar susceptibility and the in-medium condensate by writing the condensate from its expression (eq. (29)) as

$$\frac{\langle\bar qq(\rho)\rangle}{\langle\bar qq\rangle_{\rm vac}} = 1 - \frac{T(0,0)}{m_\pi^2} = \frac{1}{1 + S(0,0)/m_\pi^2}\,. \quad (33)$$

From the expression of the pseudoscalar susceptibility (eq. (26)) and using the GOR relation, one finally obtains the following relation, which is independent of the representation:

$$\chi_{\mathrm{PS}} = \frac{\langle \bar{q}q \rangle_{\mathrm{vac}}}{m_q} \frac{1}{1 + S(0,0)/m_\pi^2} = \frac{\langle \bar{q}q(\rho) \rangle}{m_q} . \qquad (34)$$

The pseudoscalar susceptibility follows the condensate evolution, *i.e.*, its magnitude decreases with density, with a linear dependence in the dilute limit where the relation $(\Sigma_A/A) = \Sigma_N$ holds. At normal density the susceptibility has thus decreased by 35%. The presence of the quark mass in the denominator of the expression of χ_{PS} makes it divergent in the chiral limit, as it should.

Our previous relation (34) between the transverse (pseudoscalar) susceptibility and the order parameter (the condensate) can be understood from the magnetic analogy. The rotational symmetry is intrinsically broken by a magnetic field $\mathbf{H}_0$ which aligns the spontaneous magnetization along its direction. The application of a small transverse field $\mathbf{H}_\perp$ rotates the magnetization $\mathbf{M}$ by a small angle θ, such that it is now aligned in the direction of the resulting field $\mathbf{H}_0 + \mathbf{H}_\perp$. The transverse magnetization is $M_\perp = M\,\theta = M(H_\perp/H_0)$ and the transverse susceptibility is $\chi_\perp = M_\perp/H_\perp = M/H_0$, which is the analog of our formula (34).

Since the pseudoscalar susceptibility is governed by the pion propagator, it is natural to discuss also its relation to the pion mass evolution [15]. We define, as usual, the in-medium effective mass, $m_\pi^*(T,\rho)$, as the energy of the pole of the pion propagator taken at zero three-momentum, which differs from the definition of Rajagopal and Wilzcek [7], where $m_\pi^{*2}(T,\rho)$ is directly taken as the inverse susceptibility. The problem of the relation between the pion mass and the condensate evolutions has already been addressed, for the nuclear medium or the heat bath [15]. In the heat bath for instance the presence of the residue in the pion propagator makes the inverse squared pion mass to decrease three times more slowly with temperature than the condensate.

A similar difference between the two evolutions occurs in the nuclear medium. The density dependence of the quark condensate, the pion propagator and the pion mass were extensively studied by Delorme *et al.* [13], up to second order in the density, in two different representations of the non-linear sigma model. They have shown that both the condensate and the pion mass are independent of the representation, as it should, the pion propagator instead is not. We reproduce in table 1 their result for these three quantities in the representation where PCAC applies and in the Weinberg representation. In the first one, the pion propagator at zero momentum and the condensate are proportional. The table clearly displays the difference between the inverse pion mass squared and the pseudoscalar susceptibility evolutions.

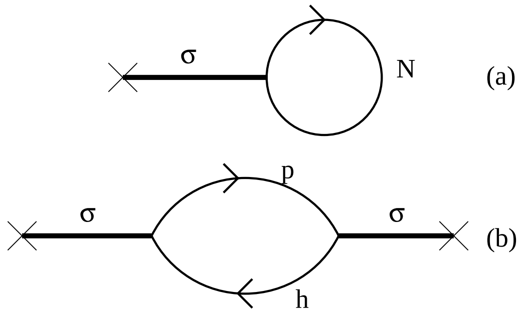

Fig. 1. Influence of the melting of the scalar field into the condensate (represented by a cross) on (a) the condensate, (b) the scalar susceptibility.

4 Results and conclusion

We now comment our conclusion that at normal density the scalar and pseudoscalar susceptibilities become closer than in the vacuum. In order to show that this convergence is not accidental but is a manifestation of chiral symmetry restoration, we have to enter into the various mechanisms responsible for the restoration process. One of them is the nuclear scalar field melting into the condensate, which has the same quantum number (see fig. 1). It leads to a modification of the condensate, $\Delta_\sigma \langle \bar{q}q(\rho) \rangle$ (accordingly of the pseudoscalar susceptibility) by

$$\Delta_\sigma \langle \bar{q}q(\rho) \rangle = -\langle \bar{q}q \rangle_{\mathrm{vac}} \frac{g_s \rho_s}{f_\pi m_\sigma^2} = B\rho_s , \qquad (35)$$

in which the second equation simply defines the proportionality factor B. Now, comparing this to the modification of the scalar suceptibility arising from the coupling of the scalar field to the nucleons, which can be written as $\Delta\chi_{\mathrm{S}} = 2\,B^2\,\Pi_{\mathrm{SS}}$, we see that the factor B enters with a square power and the nucleon density has been replaced by the nucleon-hole propagator. But a unique mechanism has produced the decrease of the order parameter and the increase of the scalar susceptibility. A similar correspondence exists for the influence of the nuclear pions. Their scalar density, which is linked to the mean value of the square pion field, $\langle \Phi^2 \rangle$, modifies the condensate according to

$$\Delta_\pi \langle \bar{q}q(\rho) \rangle = -\langle \bar{q}q \rangle_{\mathrm{vac}} \frac{\langle \Phi^2 \rangle}{2 f_\pi^2} = C\,\langle \Phi^2 \rangle . \qquad (36)$$

While the influence of the two-pion continuum on the scalar susceptibility is

$$\begin{aligned} \Delta\chi_{\mathrm{S}} &= 2\,\frac{\langle \bar{q}q \rangle_{\mathrm{vac}}^2}{f_\pi^2} \frac{1}{m_\sigma^4} \frac{m_\sigma^4}{4 f_\pi^2}\, G_{2\pi}(\mathbf{q}=0, \omega=0) \\ &= 2\,C^2\, G_{2\pi}(\mathbf{q}=0, \omega=0) . \qquad (37) \end{aligned}$$

In the scalar susceptibility the pion density is replaced by the two-pion propagator $G_{2\pi}$ and the factor C enters at the square power. The evolutions of the two susceptibilities are correlated in the sense that a unique mechanism (for instance the melting of the scalar nuclear field in the

Table 1. Comparison of the results obtained with two non-linear Lagrangians, $\mathcal{L}^{\mathrm{W}}$ which do not satisfy PCAC and $\mathcal{L}'$ which does. The successive lines give the inverse pion propagator, the squared pion effective mass and the density evolution of the quark condensate, *i.e.*, of the pseudoscalar susceptibility. We define $x = \rho\Sigma_N/f_\pi^2 m_\pi^2$ and $x_{2,3} = \rho\, c_{2,3}/f_\pi^2$, where $c_{2,3}$ are the standard parameters of the chiral Lagrangian [12].

	$\mathcal{L}^{\mathrm{W}}$	$\mathcal{L}'$
$D_\pi^{-1}(\omega,\boldsymbol{q})$	$\big[1+2(x_2+x_3)\big]\omega^2 -(1+2x_3)\boldsymbol{q}^2-(1-x)m_\pi^2$	$\Big[\big[1+2(x_2+x_3)\big]\omega^2-(1+2x_3)\boldsymbol{q}^2 -(1-x)m_\pi^2\Big]/(1-x)^2$
$\dfrac{m_\pi^{*2}}{m_\pi^2}$	$\dfrac{1-x}{1+2(x_2+x_3)}$	$\dfrac{1-x}{1+2(x_2+x_3)}$
$\dfrac{\langle\overline{q}q(\rho)\rangle}{\langle\overline{q}q(0)\rangle}$	$1-x$	$1-x$

condensate) is responsible for the enhancement of one susceptibility and the decrease of the other. In fact it is possible to establish this link by a direct evaluation of the scalar susceptibility from the condensate, by taking the derivative with respect to the quark mass. Consider for instance the contribution from the pion loops, which modifies the condensate according to eq. (36). The nuclear pion density, $\langle\Phi^2\rangle$, depends on the pion mass, *i.e.*, on the quark mass and thus generates a contribution to the nuclear scalar susceptibility. Since $\langle\Phi^2\rangle$ is related to the pion propagator, its derivative with respect to m_q is linked to the two-pion propagator taken at zero momentum, which leads to our previous eq. (37).

Coming back to the convergence of the two susceptibilities, it arises from both evolutions. The smaller relative decrease of the pseudoscalar one is compensated by the large value of this susceptibility, owing to the smallness of the pion mass. In view of this large convergence effect at normal density, it is natural to explore the phenomenon at larger densities. We cannot perform this extrapolation with certainty but we can have some indications. The mixing of the NN^{-1} states with the sigma which is responsible for the increase of the scalar susceptibility may not develop much further for several reasons. The quantity Π_{SS} involves the scalar nucleon density which increases more slowly than the ordinary density. Moreover it is proportional to the effective nucleon mass which decreases with density. Finally the nucleon reaction to the scalar field manifests itself more with increasing density. In order to evaluate the influence of these effects at any density, we take for the quantity $g_{\mathrm{S}}^2\,\tilde{D}_0^{\mathrm{S}}\,\Pi_{\mathrm{SS}}(\mathbf{q}=0,\omega=0)$ the ansatz

$$g_{\mathrm{S}}^2\,\tilde{D}_0^{\mathrm{S}}\,\Pi_{\mathrm{SS}} = g_{\mathrm{S}}\,\langle\sigma(\rho)\rangle\,\frac{3\,M_N^*(\rho)}{k_{\mathrm{F}}^2}\,, \tag{38}$$

which holds at ρ_0, since K is close to the Fermi gas value, but may not when the density increases. We take the values of the scalar field and of the effective nucleon mass from [9]. With these inputs we find that the enhancement factor of the scalar susceptibility stabilizes, with even a certain decrease. It is 5.2 at $1.5\rho_0$ and 4.8 at $2\rho_0$.

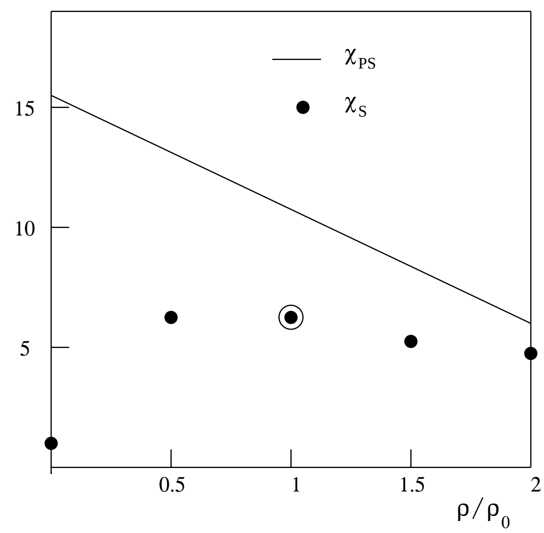

Fig. 2. Pseudoscalar (continuous line) and scalar (dots) susceptibilities as a function of the nuclear density. Both are normalized to the vacuum scalar susceptibility with a sigma mass of 550 MeV. In the pseudoscalar susceptibility the independent nucleon approximation has been assumed for the quark condensate evolution. For the scalar one, the points has been evaluated with the ansatz described in the text, but for the normal density point in which the "experimental" incompressibility has been introduced (its special character is indicated by the double circle).

The behavior with density of the two susceptibilities is shown in fig. 2 but we stress again that for the scalar one, only the point at ρ_0 rests on the experimental input of the compressibility. Moreover this evaluation only takes into account the mixing of the sigma with the nucleon-hole states. Its mixing with two-pion states should also be incorporated especially at large densities. The point at $0.5\rho_0$, evaluated with the same ansatz, is only given for illustration as one enters here in the region of the spinodal instability.

In summary we have studied two QCD susceptibilities of the nuclear medium, the scalar-isoscalar one and the pseudoscalar-isovector one. They are linked to the fluctuations of the corresponding quark densities. For the first one, the use of the linear sigma model provides a link with the propagator of the sigma-meson. In the nucleus this meson mixes with the low-lying scalar-isoscalar nuclear excitations. In this respect, it is important to stress the following point. We have assimilated the sigma field of the linear sigma model (*i.e.* the chiral partner of the pion) with the nuclear scalar field responsible of nuclear binding in QHD. However, the latter is a chiral invariant while the chiral partner of the pion is not. We have shown in a previous work [16] that the two fields can be related in the common framework of the linear sigma model. They differ by a term proportional to the pion scalar density, which has a quenching effect on the fluctuations. This is presently ignored. It will be taken into account in a forthcoming work, but we expect a moderate influence. One can also question the relevance of the sigma model for the problem of the quark density fluctuations. Actually these fluctuations couple to those of the nucleon scalar density, which increases their range. The linear sigma model provides an evaluation for this coupling which must be present. At the normal nuclear density the mixing with the nuclear excitations is constrained by standard nuclear phenomenology, *i.e.*, by the nuclear-matter incompressibility. It leads to an increase of the magnitude of the scalar susceptibility by a factor of about 6. This effect, although pronounced at normal density, does not appear to increase further with increasing density. Indeed, at higher densities, the sigma is expected to decouple from the scalar NN^{-1} excitations. As for the pseudoscalar susceptibility, which is linked to the pion propagator, we have shown that it follows the evolution of the condensate, *i.e.*, its magnitude decreases with density. The two combined effects make the scalar and pseudoscalar susceptibilities appreciably closer in the nuclear medium than in the vacuum, already at the normal density. This convergence is a strong signal of chiral symmetry restoration.

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References

1. L.S. Celenza, C.M. Shakin, W.D. Sun, J. Szweda, Phys. Rev. C **51**, 937 (1995).
2. D. Davesne, Y.J. Zhang, G. Chanfray, J. Wambach, *Proceedings of the Hirschegg 2000 Workshop*, edited by M. Buballa, W. Nörenberg, B.J. Schaefer, J. Wambach (GSI, Darmstadt, 2000) p. 88; nucl-th/0002032.
3. Z. Aouissat, R. Rapp, G. Chanfray, P. Schuck, J. Wambach, Nucl. Phys.. A **581**, 471 (1995).
4. T. Hatsuda, T. Kunihiro, H. Shimizu, Phys. Rev. Lett. **82**, 2840 (1999).
5. J. Wambach, Z. Aouissat, P. Schuck, Nucl. Phys. A **690**, 127c (2001).
6. T. Hatsuda, T. Kunihiro, Phys. Rev. Lett. **55**, 158 (1985).
7. K. Rajagopal, F. Wilczek, Nucl. Phys. B **399**, 395 (1993).
8. P.A.M. Guichon, Phys. Lett. B **200**, 235 (1988).
9. P.A.M. Guichon, K. Saito, E. Rodionov, A.W. Thomas, Nucl. Phys. A **601**, 349 (1996).
10. G. Chanfray, plenary talk at *PANIC02, September 30-October 4 2002, Osaka, Japan*, to be published in Nucl. Phys. A; nucl-th/0212085.
11. B.D. Serot, J.D. Walecka, Adv. Nucl. Phys. **16**, 1 (1986).
12. B.D. Serot, J.D. Walecka, Int. J. Mod. Phys. E **16**, 515 (1997).
13. J. Delorme, G. Chanfray, M. Ericson, Nucl. Phys. A **603**, 239 (1996).
14. M. Ericson, Phys. Lett. B **101**, 11 (1993).
15. G. Chanfray, M. Ericson, J. Wambach, Phys. Lett. B **388**, 673 (1996).
16. G. Chanfray, M. Ericson, P.A.M. Guichon, Phys. Rev. C **63**, 055202.